Contributions of $e^+e^- \to P(S)\gamma$ processes to muon $g-2$[1]

A.-Z. Dubničková[a], S. Dubnička[b], A. Liptaj[b], R. Pekárik[b]

[a] *Comenius Univ., Dept. of Theor. Physics, Faculty of Mathematics, Physics and Informatics, 842 48 Bratislava, Slovak Republic*

[b] *Inst. of Physics, Slovak Acad. of Sci., Dúbravská cesta 9,845 11 Bratislava, Slovak Republic*

Abstract

The present situation on a comparison of theoretical evaluation of the muon anomalous magnetic moment with experimental one is roughly reviewed. Then by means of a recently elaborated unitary and analytic model of the meson transition form factors the contributions of $e^+e^- \to P(S)\gamma$ processes to muon $g-2$ is estimated.

1 INTRODUCTION

The muon magnetic moment is related to the spin by the expression

$$\vec{\mu} = g\left(\frac{e}{2m_\mu}\right)\vec{s} \tag{1}$$

where the gyromagnetic ratio g is predicted theoretically (see e.g. ref. [1]) to be exactly 2. However, interactions existing in nature modify g to be slightly exceeding value 2 because of the emission and absorption of virtual photons, intermediate vector and Higgs bosons and vacuum polarization into virtual hadronic states.

In order to describe this modification of g theoretically, the magnetic anomaly was introduced by the relation

$$a_\mu \equiv \frac{g-2}{2} = a_\mu^{(1)}\left(\frac{\alpha}{\pi}\right) + \left(a_\mu^{(2)QED} + a_\mu^{(2)had}\right)\left(\frac{\alpha}{\pi}\right)^2 + a_\mu^{(2)weak} + O\left(\frac{\alpha}{\pi}\right)^3 \tag{2}$$

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where α is the fine structure constant.

The muon anomalous magnetic moment a_μ is very interesting object for theoretical and experimental investigations. It is the best measured quantity in physics [2],[3]

$$\begin{aligned} a_{\mu^+}^{exp} &= (116592040 \pm 86) \times 10^{-11} \\ a_{\mu^-}^{exp} &= (116592140 \pm 85) \times 10^{-11}. \end{aligned} \tag{3}$$

On the other hand, an accurate theoretical (better phenomenological) evaluation of a_μ^{th} provides an extremely clean test of "Electroweak theory" and any disagreement between a_μ^{th} and a_μ^{exp} may give the first window to new physics beyond the Standard Model (SM). So, just the latter pretends the evaluation of a_μ^{th} to be as precise as possible, in order to be sure that SM is taken into account with sufficiently high precision.

Nowadays the weak interaction contributions, arising from single and two-loop diagrams, are [4]-[6]

$$a_\mu^{(2,3)weak} = (152 \pm 4) \times 10^{-11}. \tag{4}$$

The QED contributions up to 8-th order give the value [7]

$$a_\mu^{QED} = (116584705.7 \pm 2.9) \times 10^{-11}. \tag{5}$$

Though T. Kinoshita recently revealed in his calculations a program error [8], the correction is not large enough to affect the comparison between theory and experiment for the muon $g-2$, but it only does alter the inferred value for the fine structure constant.

The strong interactions contribute through the Feynman diagrams presented in Figs.1-2.

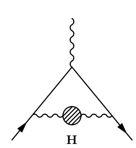

Figure 1: The lowest-order hadronic vacuum-polarization contributions.

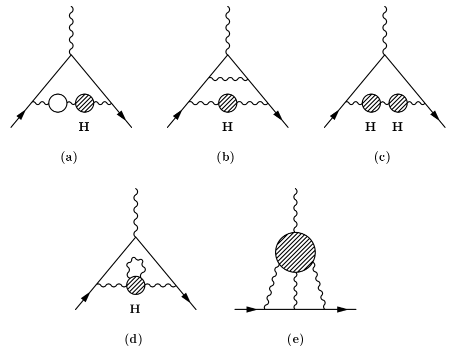

Figure 2: The third-order hadronic vacuum-polarization contributions.

The lowest-order hadronic vacuum-polarization contributions (Fig.1), according to the more recent evaluations (considering only e^+e^- annihilation cross-sections as there is still unsolved problem of disagreement [9] with τ-decay data)

$$\begin{aligned} a_\mu^{had}(l.o.) &= (6969.5 \pm 80.7) \times 10^{-11}, \quad [10] \\ a_\mu^{had}(l.o.) &= (6930.0 \pm 71.7) \times 10^{-11}, \quad [11] \\ a_\mu^{had}(l.o.) &= (6961.5 \pm 63.7) \times 10^{-11}, \quad [12] \end{aligned} \tag{6}$$

which seem to be more or less mutually consistent.

Within the recent past the most discussed from all hadronic contributions were the light–by–light (LBL) meson pole terms (see Figs.2e and 3).

More precisely, up to the end of 2001 there was a believe that the light-by-light meson-pole term contributions are acquiring negative values. However, recently it was clearly demonstrated [13], [14] that they must be positive ones, thus giving the total LBL values

$$\begin{aligned} a_\mu^{LBL} &= (80 \pm 40) \times 10^{-11} \quad [15] \\ a_\mu^{LBL} &= (111.2 \pm 21.6) \times 10^{-11} \quad [16] \\ a_\mu^{LBL} &= (136 \pm 25) \times 10^{-11} \quad [17]. \end{aligned} \tag{7}$$

The others 3-loop hadronic contributions derived from the hadronic vacuum polarizations (VP) in Figs.2(a)-(c) aquire the value [18]

$$a_\mu^{(3)VP} = (-101 \pm 6) \times 10^{-11}. \tag{8}$$

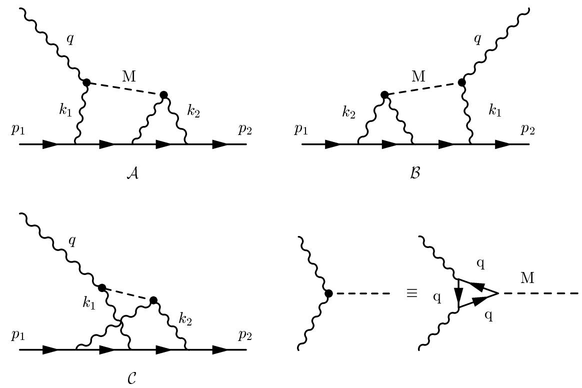

Figure 3: Meson (M) pole diagrams in the third order hadronic light–by–light scattering contributions to a_μ^{had}.

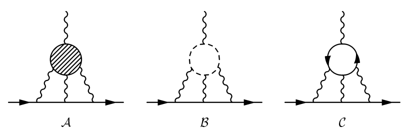

Figure 4: Third order hadronic light–by–light scattering contribution to a_μ^{had} ($\mathcal{A}$) and class of pseudoscalar meson square loop diagrams ($\mathcal{B}$) and quark square loop diagrams ($\mathcal{C}$) contributing to ($\mathcal{A}$).

If we take into account a_μ^{LBL} from [16], which is almost the average of all three ones in (7), then the total 3-loop hadronic correction is

$$a_\mu^{(3)had} = (10.2 \pm 22.4) \times 10^{-11}, \tag{9}$$

where the errors have been added in quadratures.

Now, summing all presented here contributions (4), (5), the averaged (6) and (9), finally one gets the SM theoretical prediction of the muon anomalous magnetic moment value to be

$$a_\mu^{th} = (116591832.6 \pm 82.7) \times 10^{-11}, \tag{10}$$

which in confrontation with the averaged experimentally determined value (3) gives

$$a_\mu^{exp} - a_\mu^{th} = (257.4 \pm 146.6) \times 10^{-11}. \tag{11}$$

This result might imply a window to the new physics beyond SM. Therefore it is reasonable to pay more attention to exotic channels which could diminish the difference in (11). Here we evaluate contributions of the $e^+e^- \to P(S)\gamma$ processes (with P=π^0, η, η' and S=σ, a_0), to muon $g-2$ by means of the elaborated unitary and analytic model of transition form factors (FF's) $F_{P(S)\gamma}(t)$ providing one analytic function for space-like and time-like region simultaneously.

2 $e^+e^- \to P\gamma$ PROCESSES

There is a single FF $F_{P\gamma}(t)$ for each $\gamma^* \to P\gamma$ transition completely describing a behaviour of

$$\sigma_{tot}(e^+e^- \to P\gamma) = \pi\alpha^2/2(1-m_P^2/t)^3 |\ F_{P\gamma}(t)\ |^2 \tag{12}$$

to be defined by a parametrization of the matrix element of the electromagnetic (EM) current $J_\mu^{EM} = 2/3\bar{u}\gamma_\mu u - 1/3\bar{d}\gamma_\mu d - 1/3\bar{s}\gamma_\mu s$

$$\langle P(p)\gamma(k) \mid J_\mu^{EM} \mid 0\rangle = \epsilon_{\mu\nu\alpha\beta}p^\nu\epsilon^\alpha k^\beta F_{P\gamma}(t) \tag{13}$$

where ϵ^α is the polarization vector of γ, $\epsilon_{\mu\nu\alpha\beta}$ appears as only the pseudoscalar meson belongs to the abnormal spin-parity series and $t = q^2$=$-Q^2$ is the square four-momentum transferred by the virtual photon. In the framework of the unitary and analytic model of the EM pseudoscalar-meson-transition FF's [19] $F_{P\gamma}(t)$ takes form

$$F_{P\gamma}(t) = F_{P\gamma}^{I=0}[V(t)] + F_{P\gamma}^{I=1}[W(t)] \tag{14}$$

with

$$F_{P\gamma}^{I=0}[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^2 \Big\{\frac{1}{2}F_{P\gamma}(0)H(\omega') + \\ + [L(\omega) - H(\omega')]a_\omega^P + \\ +[L(\phi) - H(\omega')]a_\phi^P\Big\}$$

$$F_{P\gamma}^{I=1}[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \Big\{\frac{1}{2}F_{P\gamma}(0)H(\rho') + [L(\rho) - H(\rho')]a_\rho^P\Big\},$$

where $V(t)(W(t))$ is a conformal mapping

$$V(t) = i\frac{\sqrt{q_{in}^{I=0}+q}-\sqrt{q_{in}^{I=0}-q}}{\sqrt{q_{in}^{I=0}+q}+\sqrt{q_{in}^{I=0}-q}} \tag{15}$$

$$q = [(t-t_0)/t_0]^{1/2}; \quad q_{in}^{I=0} = [(t_{in}^{I=0}-t_0)/t_0]^{1/2}$$

of the four-sheeted Riemann surface in t-variable into one V-plane (W-plane),

$$F_{P\gamma}(0) = \frac{2}{\alpha m_P}\sqrt{\frac{\Gamma(P\to\gamma\gamma)}{\pi m_P}} \tag{16}$$

$t_0 = m^2_{\pi^0}$, $t_{in}^{I=0}$ and $t_{in}^{I=1}$ are the effective square-root branch points including in average contributions of all higher important thresholds in both, isoscalar and isovector case, respectively,

$$L(s) = \frac{(V_N-V_s)(V_N-V_s^*)(V_N-1/V_s)(V_N-1/V_s^*)}{(V-V_s)(V-V_s^*)(V-1/V_s)(V-1/V_s^*)};$$

$$s=\omega,\phi, \quad V_N = V(t)_{|t=0}$$

$$H(\omega') = \frac{(V_N-V_{\omega'})(V_N-V_{\omega'}^*)(V_N+V_{\omega'})(V_N+V_{\omega'}^*)}{(V-V_{\omega'})(V-V_{\omega'}^*)(V+V_{\omega'})(V+V_{\omega'}^*)};$$

$$L(\rho) = \frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)};$$

$$W_N = W(t)_{|t=0}$$

$$H(\rho') = \frac{(W_N-W_{\rho'})(W_N-W_{\rho'}^*)(W_N+W_{\rho'})(W_N+W_{\rho'}^*)}{(W-W_{\rho'})(W-W_{\rho'}^*)(W+W_{\rho'})(W+W_{\rho'}^*)}$$

and

$$a_r^P = (f_{rP\gamma}/f_r); \quad r=\rho,\omega,\phi.$$

Prediction of $F_{\pi^0\gamma}(t)$, $F_{\eta\gamma}(t)$ $F_{\eta'\gamma}(t)$ behaviours and their comparison with existing data is presented in Figs. 5-7. By substitution of the latter into (12) one estimates, through the first integral in the relation

$$\begin{aligned} a_\mu^{(2)had} &= \frac{1}{4\pi^3}\left\{\int_{\pi^0}^{3GeV^2}\sum_F \sigma_{tot}(e^+e^-\to F)\times K_\mu(t)dt + \right. \\ &+ \left.\int_{3GeV^2}^{\infty} R(e^+e^-\to had)\sigma_{tot}(e^+e^-\to\mu^+\mu^-)\times K_\mu(t)dt\right\} \end{aligned} \tag{17}$$

with

$$K_\mu(t) = \int_0^1 \frac{x^2(1-x)}{x^2+(1-x)t/m_\mu^2}dx, \tag{18}$$

which represents the diagram in Fig. 1, the contributions of $e^+e^- \to P\gamma$ processes

$$\begin{aligned} a_\mu(\pi^0\gamma) &= 17.2 \times 10^{-11} \\ a_\mu(\eta\gamma) &= 2.2 \times 10^{-11} \\ a_\mu(\eta'\gamma) &= 1.5 \times 10^{-11} \end{aligned} \tag{19}$$

into the muon anomalous magnetic moment a_μ. The obtained results don't agree with the values recently evaluated in papers [20] and [21] and have to be carefully reanalyzed.

3 $e^+e^- \to S\gamma$ PROCESSES

Similarly to the unitary and analytic model of $F_{P\gamma}(t)$ FF's, one can construct the model for $F_{S\gamma}(t)$ FF's

$$F_{S\gamma}(t) = F_{S\gamma}^{I=0}[V(t)] + F_{S\gamma}^{I=1}[W(t)] \tag{20}$$

$$F_{S\gamma}^{I=0}[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^2 \left\{\frac{1}{2}F_{S\gamma}(0)H(\omega')+\right.$$

$$\left.+[L(\omega)-H(\omega')]a_\omega^S + [L(\phi)-H(\omega')]a_\phi^S\right\}$$

$$F_{S\gamma}^{I=1}[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \left\{\frac{1}{2}F_{S\gamma}(0)H(\rho') +\right.$$

$$\left.+[L(\rho)-H(\rho')]a_\rho^S\right\}$$

with the same denotation as in the case of the pseudoscalar mesons, however, here $t_0^{I=0} = 9m_\pi^2$ and $t_0^{I=1} = 4m_\pi^2$. The effective branch points $t_{in}^{I=0}$ and $t_{in}^{I=1}$ are fixed at the typical value 1GeV^2 found in a fitting of the pseudoscalar-meson transition FF data, which corresponds approximately to the $K\bar{K}$ threshold. $F_{\sigma\gamma}(0)$ value is found from the two-photon decay rate $\Gamma(\sigma \to \gamma\gamma)$=0.283 keV estimated by some of us [22] in the framework of the Nambu-Jona-Lasinio model (7), taking into account the fact that

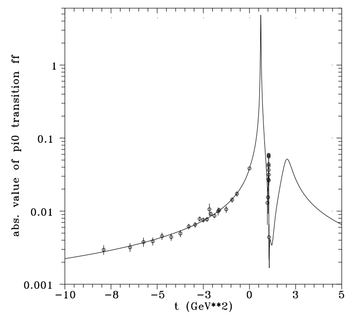

Figure 5: π^0 transition form factor.

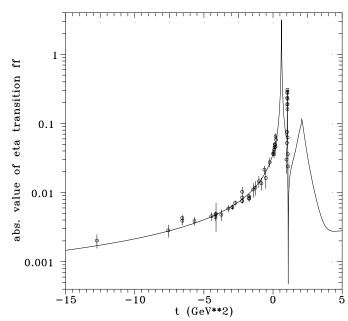

Figure 6: η transition form factor.

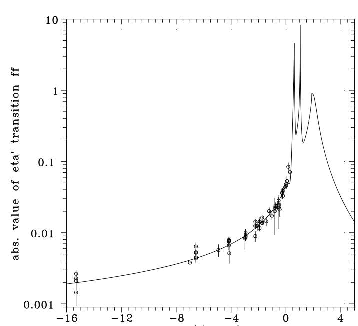

Figure 7: η' transition form factor.

Table 1: Vector meson-Scalar meson-Photon coupling constants.

g [GeV^{-1}]	QCD sum rules (s.r.)	Light cone QCD s.r	ρ photoprod.	chir.loops	$\bar{g}$
$\rho\sigma\gamma$	4.15± 0.78[24]	2.85± 0.52[27]	3.51[30]	-	3.5
$\rho a_0\gamma$	1.69± 0.39 [25]	-	-	-	1.69
$\omega\sigma\gamma$	-	0.92± 0.10 [28]	-	0.14 ± 0.01[32]	0.53
$\omega a_0\gamma$	0.57± 0.13[25]	-	-	-	0.57
$\phi\sigma\gamma$	0.042 ± 0.009[26]	0.039± 0.009[29]	0.046[31]	-	0.042
$\phi a_0\gamma$	0.12± 0.03[26]	0.13± 0.03[29]	-	-	0.125

$g_{\sigma q\bar{q}} \equiv g_{\pi^0 q\bar{q}}$. $F_{a_0\gamma}(0)$ is determined from the experimental value $\Gamma(a_0 \to \gamma\gamma)$=0.24 keV given in [23]. The coupling constant ratios a^S_r=$(f_{rS\gamma}/f_r)$, r=σ, a_0 are determined by using the averaged values $\bar{g} \equiv f_{rS\gamma}$ presented in Table 1. Then the behaviours of $|F_{S\gamma}(t)|$ are found (see dashed lines Figs. 8,9) and subsequently the contributions of $e^+e^- \to S\gamma$ processes to a_μ^{th} are evaluated $a_\mu(\sigma\gamma) = 1081.8 \times 10^{-11}$, $a_\mu(a_0\gamma) = 84.5 \times 10^{-11}$, by means of the first integral in (17).The latter clearly indicate that values of coupling constants in Table 1 are overestimated. This our hypothesis is confirmed in the case of $f_{\rho\sigma\gamma}$ by Rekalo and Tomasi-Gustafsson [33], who found the latter to be 10-times smaller than in Table 1. If we take in the lump all averaged values $\bar{g}$ of vector-meson

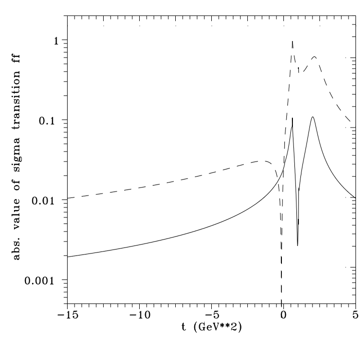

Figure 8: σ transition form factor.

scalar-meson photon coupling constants from Table 1 10-times smaller, one obtaines the behaviours of $|F_{\sigma\gamma}(t)|$ and $|F_{a_0\gamma}(t)|$ with smooth fall in the space-like region (see

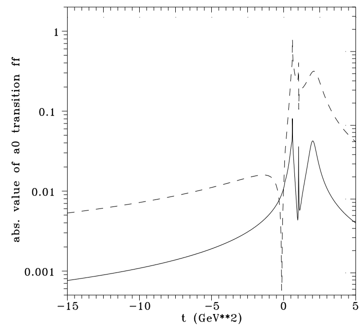

Figure 9: a_0 transition form factor.

full lines in Figs. 8,9) and quite reasonable values

$$a_\mu(\sigma\gamma) = 12.46 \times 10^{-11} \quad (21)$$
$$a_\mu(a_0\gamma) = 0.92 \times 10^{-11}$$

of $e^+e^- \to S\gamma$ contributions to a_μ^{th}.

4 CONCLUSIONS

By exploiting recently elaborated unitary and analytic model of P-meson transition FF's, we have evaluated realistic contributions of $e^+e^- \to P\gamma$ ($P = \pi^0$, η, η') processes to a_μ^{th}.

The same model was applied also to the evaluation of contributions of $e^+e^- \to S\gamma$ (S=σ, a_0) processes to a_μ^{th}. However, in this case according to our knowledge there is no experimental information on corresponding transition FF's up to now and the predicted $e^+e^- \to S\gamma$ contributions (21) to a_μ^{th} are more or less hypothetical which should be somehow justified.

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